IMA Journal of Numerical Analysis (2013) **33**, 1480–1483
doi:10.1093/imanum/drs052
Advance Access publication on May 6, 2013

Erratum to "Clenshaw–Curtis–Filon-type methods for highly oscillatory Bessel transforms and applications" (IMA Journal of Numerical Analysis (2011)31: 1281–1314)

Shuhuang Xiang
Department of Applied Mathematics and Software, Central South University, Changsha, Hunan 410083, P. R. China
*Corresponding author: xiangsh@mail.csu.edu.cn

Yeol Je Cho
Department of Mathematics Education and the RINS, Gyeongsang National University, Jinju, Korea

Haiyong Wang
School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, P. R. China
why198309@yahoo.com.cn

and

Hermann Brunner
Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NL, Canada A1C 5S7, and *Department of Mathematics, Hong Kong Baptist University, Kowloon Tong, Hong Kong*
*Corresponding author: hbrunner@math.hkbu.edu.hk

In this erratum, with respect to the formulas in the appendix of Xiang *et al.* (2011), "Clenshaw–Curtis–Filon-type methods for highly oscillatory Bessel transforms and applications" we add superscripts to the coefficients $a_j^{(s)}$ in the interpolant and correct the formulas for $d_i^{(1)}$ for $i=0,1$ and $a_{N-1}^{(2)}$.

Funding

This paper is supported partly by the National Science Foundation of China (grant No.11071260) and the Program for New Century Excellent Talents in University, State Education Ministry, China.

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Xiang, S., Cho, Y., Wang, H. & Brunner, H. (2011) Clenshaw–Curtis–Filon-type methods for highly oscillatory Bessel transforms and applications. *IMA J. Numer. Anal.* **31**, 1281–1314.

Appendix

Using the coefficients in polynomial (2.2) and setting

$$a_j^{(0)} = \begin{cases} \dfrac{\widetilde{b}_j}{2}, & j = 0, N, \\ \widetilde{b}_j, & j = 1 : N-1, \end{cases}$$

we can construct the following Hermite interpolating polynomial $p_{N+2s}(x)$ with $s = 1$ or $s = 2$ for N even:

$$p_{N+2}(x) = \sum_{j=0}^{N+2} a_j^{(1)} T_j(x) \quad \text{and} \quad \begin{cases} a_j^{(1)} = a_j^{(0)}, & j = 0 : N-3, N, \\ a_j^{(1)} = a_j^{(0)} + d_1^{(1)}, & j = N-2, \\ a_j^{(1)} = a_j^{(0)} + d_0^{(1)}, & j = N-1, \\ a_j^{(1)} = -d_0^{(1)}, & j = N+1, \\ a_j^{(1)} = -d_1^{(1)}, & j = N+2. \end{cases}$$

Here,

$$\begin{cases} d_0^{(1)} = \dfrac{1}{8N} \displaystyle\sum_{j=1}^{N} a_j^{(0)} j^2 \left(1 - (-1)^j\right) - \dfrac{1}{8N} \left(f'(1) + f'(-1)\right), \\ d_1^{(1)} = \dfrac{1}{16N} \displaystyle\sum_{j=1}^{N} a_j^{(0)} j^2 \left(1 + (-1)^j\right) - \dfrac{1}{16N} \left(f'(1) - f'(-1)\right) \end{cases} \qquad \text{(see Hasegawa, T. 2004)}$$

$$p_{N+4}(x) = \sum_{j=0}^{N+4} a_j^{(2)} T_j(x) \quad \text{and} \quad \begin{cases} a_j^{(2)} = a_j^{(1)}, & j = 0 : N-5, N, \\ a_j^{(2)} = a_j^{(1)} + \dfrac{1}{4} d_1^{(2)}, & j = N-4, \\ a_j^{(2)} = a_j^{(1)} + \dfrac{1}{4} d_0^{(2)}, & j = N-3, \\ a_j^{(2)} = a_j^{(1)} - \dfrac{1}{2} d_1^{(2)}, & j = N-2, \\ a_j^{(2)} = a_j^{(1)} - \dfrac{3}{4} d_0^{(2)}, & j = N-1, \\ a_j^{(2)} = a_j^{(1)} + \dfrac{3}{4} d_0^{(2)}, & j = N+1, \\ a_j^{(2)} = a_j^{(1)} + \dfrac{1}{2} d_1^{(2)}, & j = N+2, \\ a_j^{(2)} = -\dfrac{1}{4} d_0^{(2)}, & j = N+3, \\ a_j^{(2)} = -\dfrac{1}{4} d_1^{(2)}, & j = N+4, \end{cases}$$

where

$$\begin{cases} d_0^{(2)} = \dfrac{1}{96N}\displaystyle\sum_{j=2}^{N+2} a_j^{(1)} j^2(j^2-1)\left(1-(-1)^j\right) - \dfrac{1}{32N}\left(f''(1)-f''(-1)\right), \\ d_1^{(2)} = \dfrac{1}{192N}\displaystyle\sum_{j=2}^{N+2} a_j^{(1)} j^2(j^2-1)\left(1+(-1)^j\right) - \dfrac{1}{64N}\left(f''(1)+f''(-1)\right). \end{cases}$$

For the general case, the expression for $p_{N+2s}(x)$ can be deduced by induction on k. Supposing that

$$p_{N+2s}(x) = \sum_{j=0}^{N+2s} a_j^{(s)} T_j(x),$$

$p_{N+2(s+1)}(x)$ can be written as

$$p_{N+2(s+1)}(x) = p_{N+2s}(x) - (x^2-1)^s w_{N+1}(x)(d_0^{(s+1)} + 2d_1^{(s+1)}x),$$

where $w_{N+1}(x) = T_{N+1}(x) - T_{N-1}(x)$, $w'_{N+1}(\pm 1) = 4N$ and

$$\begin{cases} d_0^{(s+1)} = \dfrac{1}{(s+1)!2^{s+3}N}\left(f^{(s+1)}(-1) - f^{(s+1)}(1) + p_{N+2s}^{(s+1)}(1) - p_{N+2s}^{(s+1)}(-1)\right), \\ d_1^{(s+1)} = \dfrac{1}{(s+1)!2^{s+4}N}\left(p_{N+2s}^{(s+1)}(1) + p_{N+2s}^{(s+1)}(-1) - f^{(s+1)}(1) - f^{(s+1)}(-1)\right), \end{cases} \quad s \text{ is odd};$$

$$\begin{cases} d_0^{(s+1)} = \dfrac{1}{(s+1)!2^{s+3}N}\left(p_{N+2s}^{(s+1)}(1) + p_{N+2s}^{(s+1)}(-1) - f^{(s+1)}(1) - f^{(s+1)}(-1)\right), \\ d_1^{(s+1)} = \dfrac{1}{(s+1)!2^{s+4}N}\left(f^{(s+1)}(-1) - f^{(s+1)}(1) + p_{N+2s}^{(s+1)}(1) - p_{N+2s}^{(s+1)}(-1)\right), \end{cases} \quad s \text{ is even}.$$

Applying Mason & Handscomb (2003, (2.39) and (2.41)),

$$xT_j(x) = \frac{1}{2}\left(T_{j+1}(x) + T_{|j-1|}(x)\right), \quad (1-x^2)T_j(x) = -\frac{1}{4}\left(T_{j+2}(x) - 2T_j(x) + T_{|j-2|}(x)\right),$$

we can derive $p_{N+2(s+1)}(x) = \sum_{j=0}^{N+2(s+1)} a_j^{(s+1)} T_j(x)$.

When N is odd, similar formulas can be induced by using

$$w'_{N+1}(1) = 4N, \quad w'_{N+1}(-1) = -4N.$$

For example, in the cases $s=1$ and $s=2$, the formulas for $a_j^{(s)}$ are still true by using

$$\begin{cases} d_0^{(1)} = \dfrac{1}{8N}\displaystyle\sum_{j=1}^{N} a_j^{(0)} j^2\left(1+(-1)^j\right) - \dfrac{1}{8N}\left(f'(1)-f'(-1)\right), \\ d_1^{(1)} = \dfrac{1}{16N}\displaystyle\sum_{j=1}^{N} a_j^{(0)} j^2\left(1-(-1)^j\right) - \dfrac{1}{16N}\left(f'(1)+f'(-1)\right), \end{cases}$$

and

$$\begin{cases} d_0^{(2)} = \dfrac{1}{96N} \displaystyle\sum_{j=2}^{N+2} a_j^{(1)} j^2 (j^2 - 1) \left(1 + (-1)^j\right) - \dfrac{1}{32N} \left(f''(1) + f''(-1)\right), \\ d_1^{(2)} = \dfrac{1}{192N} \displaystyle\sum_{j=2}^{N+2} a_j^{(1)} j^2 (j^2 - 1) \left(1 - (-1)^j\right) - \dfrac{1}{64N} \left(f''(1) - f''(-1)\right), \end{cases}$$

instead of $d_i^{(s)} \, (i = 0, 1)$ when N is even.